

Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2) \text{ cm}$, $BC = (3x + 10) \text{ cm}$, $AC = 7x \text{ cm}$, angle $BAC = 60^\circ$ and angle $ACB = \theta^\circ$

(a) (i) Show that $17x^2 - 35x - 48 = 0$

(3)

(ii) Hence find the value of x .

(1)

(b) Hence find the value of θ giving your answer to one decimal place.

(2)

(a) Show that the equation

$$4 \tan x = 5 \cos x$$

can be written as

$$5 \sin^2 x + 4 \sin x - 5 = 0$$

(3)

(b) Hence solve, for $0 < x \leq 360^\circ$

$$4 \tan x = 5 \cos x$$

giving your answers to one decimal place

(4)

(c) Hence find the **number of solutions** of the equation

$$4 \tan 3x = 5 \cos 3x$$

in the interval $0 < x \leq 1800^\circ$, explaining briefly the reason for your answer.

(2)

(a) Sketch the graph of $y = \sin(x - 30)$ for x in the interval $0 \leq x < 360^\circ$

(2)

(b) Find all values for x in the interval $0 \leq x < 360^\circ$, for which

$$\sin(x - 30) = 0.3$$

(4)

Give your answers to one decimal place.

4) (i) To get an equation, we can use cosine rule since we have one angle with all 3 sides.

$$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos \angle BAC$$

$$(3x+10)^2 = (x+2)^2 + (7x)^2 - \cancel{2(x+2)(7x)} \cos 60^\circ \quad (1)$$

$$9x^2 + 60x + 100 = x^2 + 4x + 4 + 49x^2 - 7x^2 - 14x \quad (1)$$

$$9x^2 + 60x + 100 = 43x^2 - 10x + 4$$

$$34x^2 - 70x - 96 = 0$$

$$\therefore 17x^2 - 35x - 48 = 0 \quad (1)$$

$$(ii) \quad 17x^2 - 35x - 48 = (17x + 16)(x - 3) = 0$$

$$x = -\frac{16}{17}, \quad x = 3 \quad (1)$$

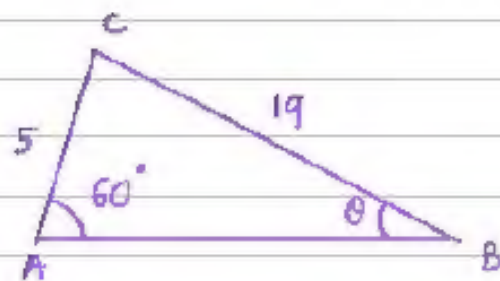
length cannot be negative value

\therefore Since x can only be positive, $x = 3$ is the only solution.

b) when $x = 3$,

$$AB = (x+2) \text{ cm} = 5 \text{ cm}$$

$$BC = (3x+10) \text{ cm} = 19 \text{ cm}$$



using sin rule to get the angle θ :

$$\frac{\sin \theta}{5 \text{ cm}} = \frac{\sin 60^\circ}{19 \text{ cm}} \quad (1)$$

$$\sin \theta = \frac{5}{19} \sin 60^\circ$$

$$\theta = \sin^{-1} \frac{5\sqrt{3}}{38} \quad (1)$$

$$= 13.17^\circ = 13.2^\circ \text{ (1 d.p.)} \quad (1)$$

$$a) 4 \tan x = 5 \cos x$$

$$4 \frac{\sin x}{\cos x} = 5 \cos x \quad (1)$$

$$4 \sin x = 5 \cos^2 x$$

$$4 \sin x = 5 (1 - \sin^2 x) \quad (1)$$

$$4 \sin x = 5 - 5 \sin^2 x$$

$$5 \sin^2 x + 4 \sin x - 5 = 0 \quad (1)$$

$$b) 4 \tan x = 5 \cos x \Rightarrow 5 \sin^2 x + 4 \sin x - 5 = 0 \quad (1)$$

$$\text{Let } a = \sin x, \quad 5a^2 + 4a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{29}}{5}$$

$$\sin x = \frac{-2 \pm \sqrt{29}}{5} \quad (1)$$

$$\boxed{\sin x = \frac{-2 + \sqrt{29}}{5}}, \quad \frac{-2 - \sqrt{29}}{5}$$

not a solution
because the value is less than -1, so it is outside of range of $\sin x$

$$x = \sin^{-1} \frac{-2 + \sqrt{29}}{5} \quad (1)$$

$$= 42.6^\circ \text{ and } 180^\circ - 42.6^\circ$$

$$(1) = 137.4^\circ$$

take the value from 2nd quadrant also since \sin is true on this quadrant.

sin	ALL
cos	1st
tan	3rd
cot	4th
sec	cos

c) for interval of $0 < x \leq 360^\circ$,

The number of solution would be $2 \times 3 = 6$ because the
 x value decrease by factor of 3. ①

for interval of $0 < x \leq 1800$, which is 5 times larger than the
previous interval,

the number of solutions would be $6 \times 5 = 30$. ①

